General Certificate of Education
June 2006
Advanced Subsidiary Examination

**MATHEMATICS** **MFP1**
**Unit Further Pure 1**

Monday 12 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

## Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

## Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

## Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots $\alpha$ and $\beta$.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots $\alpha^3$ and $\beta^3$, giving your answer in the form $px^2 + qx + r = 0$, where $p$, $q$ and $r$ are integers. *(3 marks)*

**2** A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of $y$ at $x = 2.4$. Give your answer to three decimal places. *(6 marks)*

**3** Show that

$$\sum_{r=1}^{n} (r^2 - r) = kn(n+1)(n-1)$$

where $k$ is a rational number. *(4 marks)*

**4** Find, in **radians**, the general solution of the equation

$$\cos 3x = \frac{\sqrt{3}}{2}$$

giving your answers in terms of $\pi$. *(5 marks)*

**5** The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by $\mathbf{M}$. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

**6** It is given that $z = x + \mathrm{i}y$, where $x$ and $y$ are real numbers.

(a) Write down, in terms of $x$ and $y$, an expression for

$$(z + \mathrm{i})^*$$

where $(z + \mathrm{i})^*$ denotes the complex conjugate of $(z + \mathrm{i})$. *(2 marks)*

(b) Solve the equation

$$(z + \mathrm{i})^* = 2\mathrm{i}z + 1$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

**Turn over for the next question**

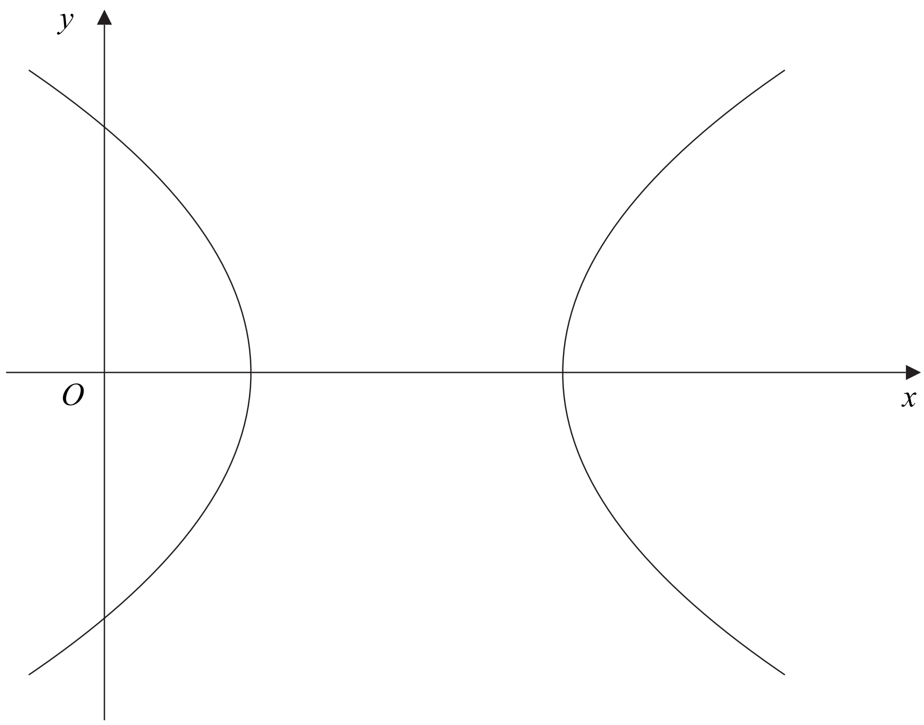
**7** (a) Describe a geometrical transformation by which the hyperbola

$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola $H$ with equation

$$x^2 - y^2 - 4x + 3 = 0$$




By completing the square, describe a geometrical transformation by which the hyperbola $H$ can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*

**8** (a) The function f is defined for all real values of $x$ by

$$f(x) = x^3 + x^2 - 1$$

(i) Express $f(1+h) - f(1)$ in the form

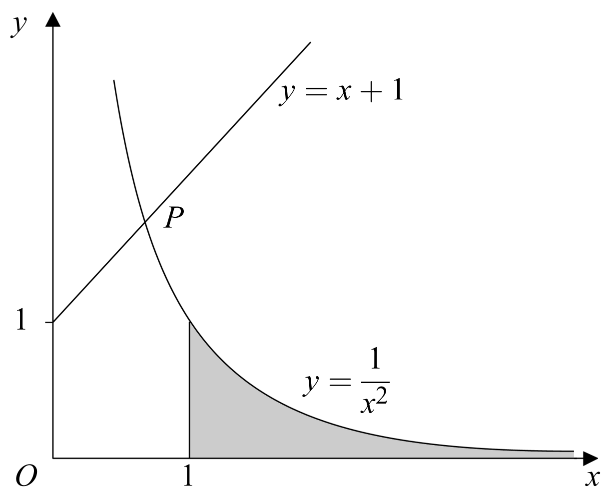
$$ph + qh^2 + rh^3$$

where $p$, $q$ and $r$ are integers. *(4 marks)*

(ii) Use your answer to part (a)(i) to find the value of $f'(1)$. *(2 marks)*

(b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$




The graphs intersect at the point $P$.

(i) Show that the $x$-coordinate of $P$ satisfies the equation $f(x) = 0$, where f is the function defined in part (a). *(1 mark)*

(ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $f(x) = 0$, use the Newton–Raphson method to find a second approximation $x_2$ to the root. *(3 marks)*

(c) The region enclosed by the curve $y = \dfrac{1}{x^2}$, the line $x = 1$ and the $x$-axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. *(3 marks)*

**9** A curve $C$ has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

(a) (i) Write down the coordinates of the points where $C$ intersects the $x$-axis. *(2 marks)*

(ii) Write down the equations of all the asymptotes of $C$. *(3 marks)*

(b) (i) Show that, if the line $y = k$ intersects $C$, then

$$(k-1)(k-4) \geqslant 0$$ *(5 marks)*

(ii) Given that there is only one stationary point on $C$, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(3 marks)*

(c) Sketch the curve $C$. *(3 marks)*

**END OF QUESTIONS**

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